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13 ABSTRACT

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14 The formation of vapor bubbles in a metastable liquid, cavitation, is an activated process due to the free energy cost of having both phases at 15 contact. Such an energetic penalty enables the existence of the liquid beyond its thermodynamic borders. Establishing the stability limits of a 16 liquid as ubiquitous as water has important practical implications and has thereby attracted a lot of attention. Different experimental strategies 17 and theoretical analyses have been employed to measure and predict the cavitation line, or the pressure-temperature kinetic stability border of 18 liquid water. Understanding the location of the cavitation line requires knowing the cavitation rate dependence on pressure and temperature. 19 Such dependency is difficult to obtain in experiments, and we use molecular simulations with the TIP4P/2005 model to fill this gap. By deeply 20 overstretching liquid water below the saturation pressure, we are able to observe and quantify spontaneous cavitation. To deal with a lower overstretching regime, we resort to the Seeding technique, which consists of analyzing simulations of a liquid containing a vapor bubble 21 22 under the theoretical framework of Classical Nucleation Theory. Combining spontaneous cavitation with Seeding, we get a wide overview of the cavitation rate along with pressure. We study two different temperatures (450 and 550 K) and complement our perspective with the results 23 24 previously obtained at 296.4 K [Menzl et al., Proc. Natl. Acad. Sci. 113, 13582 (2016)] to establish a broad simulation-experiment comparison. 25 We find a good agreement between simulations and both isobaric heating and isochoric cooling experiments using quartz inclusions. We are, 26 however, unable to reconcile simulations with other experimental techniques. Our results predict a decrease in the solid-liquid interfacial free energy as the liquid becomes increasingly overstretched with a temperature independent Tolman length of 0.1 nm. Therefore, the capillarity 27 28 approximation underestimates the nucleation rate. Nonetheless, it provides a fair indication of the location of the cavitation line given the steep rate vs pressure dependence. Overall, our work provides a comprehensive view of the water cavitation phenomenon and sets an efficient 29 30 strategy to investigate it with molecular simulations.

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33 I. INTRODUCTION

34 Many microwave ovens display a sticker with a symbol rec-35 ommending placing a spoon in the mug where water is heated. 36 The spoon acts as a surface to aid the emergence of vapor bubbles. 37 When bubbles are formed, a liquid-vapor equilibrium is established, 38 and heat transferred by microwaves is employed in increasing the 39 number of molecules in the vapor phase rather than in a further tem-40 perature raise. In the absence of a spoon, however, bubbles may not 41 form, and the liquid may get superheated beyond the boiling tem-42 perature. In such cases, water can boil explosively when handled by

the user. Superheating is not the only manner to have metastable liquid water with respect to the vapor. Metastability can also be achieved by overstretching the liquid below the vapor saturation pressure. This route enables probing negative pressures, which is of great interest for the study of the thermodynamic behavior of liquid water and the understanding of its anomalies.^{1–4} The emergence of vapor cavities in the metastable liquid (cavitation) sets the limit for the superheating or the overstretching that the liquid can sustain. Cavitation plays an important role in many industrial, technological, and geological processes,^{5–11} which makes the understanding of the physics that governs it of paramount importance.

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This interest has prompted numerous experimental studies of 54 55 water cavitation where vapor bubbles arise upon isobaric heating,¹² application of acoustic waves,^{13,14} or isochoric cooling.^{1,15–18} In these 56 57 experiments, the metastability limit of the liquid with respect to the vapor is determined. The boundary between the metastable liquid 58 and the vapor is a cavitation line in the pressure-temperature plane 59 below which bubbles readily nucleate.^{1,19,20} This cavitation line is 60 61 analogous to the so-called homogeneous ice nucleation line,²¹ which 62 sets the temperature boundary (along pressure) for the metastabil-63 ity of the liquid with respect to ice. In both metastability situations 64 (liquid vs vapor and liquid vs solid), it is the unfavorable interfacial free energy between the emerging phase (vapor or ice) and the 65 66 surrounding liquid that enables the survival of the latter beyond its thermodynamic stability limits.^{22,23} 67

Experimental investigations on cavitation have been comple-68 69 mented by theoretical work in the framework of Classical Nucleation Theory (CNT)^{2,24,25} or, more recently, Density Functional Theory,²⁰ 70 71 with the aim of predicting and understanding the location of the 72 cavitation line. The agreement between theoretical predictions and experiments is quite satisfactory^{1,19,20} if the interfacial free energy 73 between the liquid and the vapor is allowed to decrease when moving 74 75 away from coexistence.2,

76 An important drawback of both theory and experiments is 77 that they do not have access to detailed information at the molec-78 ular scale. For instance, the critical bubble (that has a 50% chance 79 of either growing or redissolving) cannot be visualized experimen-80 tally due to its small size and short lifetime. Computer simulations can bridge this gap, and they have been extensively used to study 81 cavitation in model systems like the Lennard-Jones fluid.²⁶⁻³³ Sim-82 83 ulation work for water cavitation is more scarce, although due to the 84 technical difficulty of studying a rare event like cavitation-which 85 requires the use of special simulation methods-in a costly system 86 from a computational point of view due to the presence of electro-87 static interactions. Reference 3 pioneers in the simulation work on 88 water cavitation. With the use of biased simulations that promoted 89 the appearance of bubbles in the overstretched liquid, the authors of 90 Ref. 3 were able to calculate the nucleation rate vs the liquid pres-91 sure for temperature T = 296.4 K. The nucleation rate—the number 92 of critical bubbles that appear per unit of time and volume-is the 93 central parameter in nucleation. By knowing the nucleation rate, one 94 can estimate the pressure at which cavitation is going to take place in 95 a certain experimental setup, which means that one can predict the 96 location of the cavitation line. In fact, the authors of Ref. 3 report a 97 cavitation pressure of -126 MPa at the studied temperature and for 98 the selected water model (TIP4P/2005³⁴), which is in line with the 99 measurements obtained in isochoric cooling in quartz inclusions,¹ 100 although at odds with acoustic cavitation experiments.

101 In this paper, we revisit the cavitation of overstretched water 102 with simulations using the TIP4P/2005 water model.³⁴ We focus 103 on two different temperatures as compared to Ref. 3, namely 450 104 and 550 K, enabling a more direct comparison with the regime 105 where most experimental points of the homogeneous cavitation 106 line are available. Moreover, we use an efficient computational 107 approach based on directly simulating the liquid with a vapor 108 bubble already formed at the beginning of the simulations. This strategy, called Seeding,^{29,42-} 109 combines average properties 110 obtained in the simulations of the vapor bubbles with the theoret-111 ical formalism of CNT.^{4/-49} We complement Seeding with unbiased "brute force" molecular simulations in a high overstretching regime where bubbles appear spontaneously. Although such a regime is not attainable in experiments, we use brute force results to validate Seeding simulations that do overlap with experimentally relevant conditions.

Our simulation results for the location of the homogeneous cavitation line compare satisfactorily with theoretical predictions^{20,25} and experimental measurements.^{1,15} Simulated bubbles have an interfacial thickness of about 1 nm for 450 K and 2 nm for 550 K. The equi-density²⁸ definition of the bubble radius gives a better consistency between Seeding and Brute Force simulations than the Gibbs equi-molar surface. The interfacial free energy decreases as vapor bubbles become smaller (or as pressure decreases), which is inferred from our simulations. We also find that the vapor bubble pressure obtained by equating its chemical potential to that of the surrounding liquid is consistent with that given by the density in the interior of the bubble. This result, although expected, is not verified in liquid-to-solid phase transitions. Overall, our work shows that water cavitation can be studied from a molecular perspective, giving experimentally relevant information in a rather efficient manner.

II. SIMULATION DETAILS

We use the TIP4P/2005 force field for water, which is a rigid model consisting of three fixed point charges and one Lennard–Jones (LJ) center.³⁴

Molecular dynamics simulations are performed with the GROMACS package 2018 (single precision).⁵³ When simulating in the NpT ensemble, we use the Parrinhello-Rahman barostat,⁵⁴ with a relaxation time of 2 ps and a compressibility of 10^{-5} bar⁻¹. To control the temperature, we use the Nosé-Hoover thermostat⁵⁵ with a relaxation time of 2 ps. For the Verlet integration of the equations of motion, we use a time step of 2 fs. To deal with electrostatic interactions, we use particle mesh Ewald summations.⁵⁶ The cut-off radius for the neighbor list, the dispersive Lennard-Jones interactions, and the real part of the electrostatic interactions are 13 Å. Ewald summations have been previously used to deal with heterogeneous systems,⁵⁷ including water bubbles.^{3,58} We have checked for specific cases of bubble simulations that the energy barely changes by increasing the Ewald cutoff to 20 Å. Moreover, we find that even using a plain 20 Å cutoff for the electrostatic interactions does not almost change the average energy. These arguments give us confidence in the use of Ewald summations with the selected 13 Å real part cutoff. No tail corrections are added to the Lennard-Jones potential due to the presence of two phases with strongly different densities in our simulations. We use a switch radius of 12 Å, meaning that the LJ potential is not modified up to this radius, after which it is gradually switched off to reach zero at the cut-off radius. The LINCS algorithm is used to fix the geometry of the water molecules, with an order of six and four iterations.

III. RESULTS

Most results discussed below correspond to a temperature of 450 K. Only at the end, in Sec. III B 5, we add results corresponding to 550 K in order to establish a broader comparison with experiments.

167 The liquid-vapor equilibrium for this model has been charac-168 terized in Ref. 61. We recalculate here the coexistence pressure at 169 T = 450 K to both test our simulation setup and to account for the 170 use of a slightly different cutoff radius. We first equilibrate a liquid of 171 4096 water molecules in a cubic box in the NpT ensemble. After, we 172 introduce some empty space next to the liquid, finishing with a box 173 of $3.91724 \times 3.91724 \times 27.14$ nm³. We run this system (in the NVT 174 ensemble) until equilibrium has been reached and obtain the pressure components of the system. The pressure component normal to 175 176 the vapor-liquid interface is the vapor (or coexistence) pressure, and 177 it does not depend on the system size; in our case, $p_z = 4.49$ bar, in 178 very good agreement with the values reported in Ref. 61 (4.46 bars) 179 and Ref. 62 (4.5 bars). The tangential components of the pressure 180 depend on the system size and in our case are $p_x = -24.38$ bars and $p_{y} = -24.48$ bars. The interfacial free energy at coexistence, γ_{c} , is 181 obtained using the mechanical route,63 182

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$$\gamma_c = \frac{L_z \left[p_z - \frac{p_x + p_y}{2} \right]}{2}.$$
 (1)

¹⁸⁵ We obtain $\gamma_c = 40(1)$ mJ m⁻², which is similar to the value ¹⁸⁶ calculated in Ref. 62, $\gamma_c = 38.8$ mJ m⁻².

187 B. Cavitation rate

1. Cavitation rate by brute force molecular dynamics simulations

190 By lowering the pressure of the liquid below the coexistence 191 value, one should observe the nucleation of vapor cavities (cavitation). Owing to the liquid-vapor interfacial free energy, this process 192 193 is activated and can take a long time. At very low pressures, how-194 ever, the thermodynamic drive for the emergence of the vapor phase 195 is high, and the cavitation process is relatively quick. In fact, it is pos-196 sible to observe it in brute force molecular dynamics simulations of the bulk liquid for negative pressures below -650 bar. This is shown 197 198 in Fig. 1, where we plot the volume of the simulation box vs time 199 for ten different trajectories starting from a bulk liquid consisting of 200 20 000 water molecules under periodic boundary conditions at 450 K and -700 bar. In all runs, the system abruptly increases its volume 201 202 after some induction period (which varies stochastically from one 203 trajectory to another) due to the sudden appearance and growth of a 204 vapor bubble. The stochastic character of cavitation is a consequence 205 of its activated nature. The fact that there is a long induction period 206 indicates that a single bubble is formed in the system (an unlikely 207 event does not happen twice or more at the same time). This enables 208 us to estimate the bubble nucleation rate as

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$$J = \frac{1}{\langle t \rangle \langle V \rangle},$$

2)

210 using 1 in the numerator because one only bubble forms in each run. 211 $\langle t \rangle$ is the average time that it takes to observe the nucleation of a bub-212 ble, which is obtained as the arithmetic average between the times at 213 which the volume sharply shoots up in our ten trajectories. $\langle V \rangle$ is the 214 average volume of the liquid where bubble nucleation takes place. In 215 Table I, we report the nucleation rates for different pressures along-216 side the variables required to compute them. Obviously, the lower 217

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FIG. 1. Volume of the simulation box vs time starting from ten liquid configurations containing 20 000 molecules. The thermodynamic state point is 450 K and -700 bar.

the pressure, the higher the cavitation rate. With our brute force calculations, we span three orders of magnitude of the nucleation rate: from 10^{33} m⁻³ s⁻¹ at -750 bar to 10^{30} m⁻³ s⁻¹ at -650 bar. Probing lower nucleation rates with brute force calculation is too demanding computationally. It is important, however, to get estimates of lower nucleation rates given that in experiments one has access to rates of the order of Ref. 20 10^{15} m⁻³ s⁻¹. To do that, we resort to the Seeding technique, which is described in Sec. III B 2.

2. Cavitation rate via NVT Seeding

We shall compute the nucleation rate for higher pressures (lower rates) using Seeding.^{42–46} In this method, instead of waiting for a bubble to spontaneously appear, the simulation is started from a configuration of a liquid containing a bubble. When equilibrated in the *NVT* ensemble, the bubble turns out to be the critical vapor nucleus at the simulation temperature and at the pressure acquired by the surrounding liquid.²⁹ With such pressure and the bubble radius, one can obtain an estimate of the nucleation rate using CNT.²⁹

To prepare the initial configuration, we first equilibrate liquid water in the NpT ensemble at 450 K and a selected pressure below saturation. Next, we remove the molecules inside a sphere in the center of the box. This system is then simulated in the *NVT* ensemble and allowed to equilibrate during 1 ns. We repeat this process with different pressures, box sizes, and cavity radii looking for trajectories having a stable vapor bubble (stabilizing spherical nuclei in the *NVT* ensemble is not always viable and depends on a subtle balance between the size of the box and that of the nucleus^{29,51}). In Table II, we summarize the system sizes employed in our Seeding simulations

TABLE I. Data leading to the calculation of J via brute force molecular dynamics simulations using a system of 20 000 TIP4P/2005 water molecules at 450 K.

	3		
p (bar)	$\langle t \rangle$ (ns)	$\langle V \rangle (\mathrm{nm}^3)$	$J(s^{-1} m^{-3})$
-750	0.5(2)	747.8(1)	$3(1) \times 10^{33}$
-725	1.1(2)	743.73(6)	$1.2(2) \times 10^{33}$
-700	7(3)	739.81(3)	$2.0(8) \times 10^{32}$
-675	40(10)	736.15(1)	$3.1(8) \times 10^{31}$
-650	300(100)	732.798(7)	$4(2) \times 10^{30}$

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ARTICLE

T (K)	Ν	L (nm)	p_l (bar)	Δp (bar)	$\rho_l (\mathrm{kg}/\mathrm{m}^3)$	$\rho_{\nu} (\mathrm{kg/m^3})$	R _{ed} (nm)	R_G (nm)	$\log_{10}[J_{ed}/({\rm s}^{-1}~{\rm m}^{-3})]$	$\log_{10}[J_G/(s^{-1} m^{-3})]$
	58 817	13.9305	-135.4	142.2	869.8	3.79	5.43	5.46	-293	-299
64 085	13.9305	-150.4	156.9	868.6	3.65	4.88	4.92	-227	-233	
	47 110	12.3315	-190.8	196.8	865.2	3.34	3.85	3.89	-124	-130
49 774 12.3315 49 774 12.3315	12.3315	-225.5	231.2	862.2	3.13	3.24	3.29	-75	-80	
	12.3315	-226.2	231.9	862.2	3.12	3.24	3.29	-75	-80	
450	⁴⁵⁰ 51 785 12.1 17 985 8.1	12.3315	-295.4	300.6	856.0	2.79	2.45	2.50	-24	-29
		8.8216	-308.7	313.7	854.8	2.73	2.33	2.39	-17	-22
18 548 18 524 8 950	8.8056	-391.0	395.6	847.0	2.44	1.80	1.88	7	2	
	8.8023	-391.0	395.6	847.0	2.44	1.80	1.88	7	2	
	6.9651	-435.5	439.8	842.6	2.31	1.58	1.69	15	9	
	50 691	13.1049	-73.2	110.9	710.2	20.92	2.93	3.05	7	2
550 17 064 17 737	9.2936	-86.4	123.7	706.6	20.56	2.55	2.70	16	11	
	9.2936	-116.4	152.8	697.8	19.78	1.99	2.18	26	21	

259 TABLE II. Data corresponding to the different NVT Seeding simulations performed in this work. See the main text for the meaning of the different reported parameters

277 alongside the radii of the stabilized bubbles averaged over 19 ns. 278 We report two different radii for each bubble. One is R_G , the radius 279 corresponding to the equi-molar Gibbs dividing surface, which is 280 computed as follows:

$$R_{G} = \left(\frac{3(N - L^{3}\rho_{l})}{4\pi(\rho_{v} - \rho_{l})}\right)^{1/3},$$
(3)

283 where N is the total number of molecules, L is the length of the simu-284 lation box edge (since we work with a cubic system the volume of the 285 box is L^3), and ρ_l and ρ_v are the densities of the liquid and the vapor 286 phases, respectively. The former is obtained via the average virial 287 pressure, p_l , and the bulk liquid equation of state (see Table II). The 288 latter is obtained by looking for the vapor that has the same chemical potential as the surrounding liquid: $^{64,65} \mu_l(T, p_l) = \mu_v(T, p_v)$. We 289 search where this condition is satisfied by performing isothermal 290 291 thermodynamic integration of both the bulk vapor and the bulk liq-292 uid molar volumes from the coexistence pressure, p_{coex} , as described

u(p)-μ(p_{coex}) (kcal mol⁻¹) Vapour - Liquid Coexistence -0.1 -450 -350 p -150 -50 p_v -25050 p (bar)

FIG. 2. Chemical potential difference with respect to the coexistence pressure at 293 294 450 K for the bulk liquid (black) and the bulk vapor (red) phases. Dashed and dotted horizontal and vertical lines guide the search for the condition of equal chemical 295 296 potential in a bubble of radius ~2.5 nm (see main text for more details).

in Ref. 28. In Fig. 2, we show $\mu(p) - \mu(p_{coex})$ at 450 K for the liquid (black curve) and the vapor (red curve). The dotted vertical line at p = -295.4 bar corresponds to p_l for a bubble of 2.5 nm. The chemical potential of such liquid can be obtained from the intersection between the dotted line and the black curve. Then, we read from the intersection between the horizontal dashed line and the red curve, the pressure of the vapor that has the same chemical potential as the liquid, p_{v} . Δp , a key parameter in the CNT nucleation formalism, is the difference between p_v and p_l . The vapor pressures, p_v , and the corresponding densities, ρ_{ν} , thus obtained for all simulated bubbles, are reported in Table II.

Alternatively, we compute the bubble radius by means of an average radial density profile starting from the bubble center, as shown in Fig. 3. Such a density profile is fitted to the following sigmoid function (dashed orange curve in Fig. 3):

$$\rho(r) = \frac{\rho_{\nu,dp} + \rho_{l,dp}}{2} + \left(\frac{\rho_{l,dp} - \rho_{\nu,dp}}{2}\right) \cdot \tanh[(r - R_{ed})/\alpha], \quad (4) \quad {}^{312}$$



FIG. 3. Radial density profile starting from the bubble center averaged over 19 ns in the NVT ensemble (black line). The profile corresponds to a bubble surrounded by a liquid of pressure -295.4 bar. The dashed orange line is a fit to the simulation data given by Eq. (4).

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where $\rho_{v,dp}$ and $\rho_{l,dp}$ are the densities of the vapor and the liquid phases obtained with the density profile (dp) fit, α is a parameter related to the width of the interfacial region, and R_{ed} is the critical bubble radius estimated as the distance at which the density is the average between both phases ("equi-density" criterion).²⁸ The equidensity radius, R_{ed} , reported in Table II is close to but systematically lower than R_G .

Both approaches obtain the radius [Eqs. (3) and (4)] assume a spherical bubble shape. In order to visually inspect the shape, we provide, in Fig. 4, a series of slabs cut through the bubble center at different times along a Seeding simulation containing a 2.5 nm bubble. The sequence of snapshots clearly shows that the bubble shape fluctuates around a sphere, which justifies our approach to obtain the bubble radius assuming an average spherical shape.

The fit parameter in Eq. (4) $\rho_{l,dp}$ is fully consistent with the den-331 332 sity obtained by means of the overall virial pressure of the system and 333 the bulk liquid equation of state. However, the vapor density provided by the fit given by Eq. (4), $\rho_{v,dp}$, is not accurate, given the small 334 335 size of the bubbles (the sigmoid does not reach a true plateau in the 336 interior of the bubble). Nevertheless, by estimating the vapor den-337 sity as an average of the density profile in a region in the interior of 338 the bubble, one does obtain consistent values with the vapor density 339 resulting by imposing equal chemical potential between the bubble and the surrounding liquid. This consistency, although expected, 340 341 should not be given for granted. For instance, for hard sphere crys-342 tallization, the density (and the mechanical pressure) of the crystal 343 nucleus is lower than that obtained by imposing chemical poten-344 tial homogeneity (in fact, the nucleus's mechanical pressure is even 345 lower than that of the surrounding liquid).

³⁴⁶ Knowing the bubble radius and Δp , we can use CNT to estimate the nucleation rate, *J*,

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$$J = A \, \exp\left(-\frac{\Delta G_c}{k_B T}\right),\tag{5}$$

where k_B is the Boltzmann constant, A is a kinetic pre-factor, and ΔG_c is the Gibbs free energy associated with the formation of the critical bubble, which is given by



FIG. 4. Slab cuts of 20 Å thickness through the bubble center in a Seeding simulation at 450 K containing a ~2.5 nm radius sphere.

$$\Delta G_c = \frac{2\pi \times \Delta p \times R_c^3}{3},\tag{6}$$

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where R_c is the critical bubble radius that we identify either with R_G or with R_{ed} . The kinetic pre-factor is computed following Blander and Katz:²⁴

$$A = \sqrt{\frac{\Delta p \times R_c}{\pi m}} \rho_l, \tag{7}$$

where *m* is the mass of the molecule (18.02 u for water). This theoretical expression gives consistent results with other routes to estimate the kinetic pre-factor.⁶⁶ One can also obtain from the bubble properties an estimate of the interfacial free energy via the Laplace equation,

$$\gamma = \frac{\Delta p \times R_c}{2}.$$
 (8)

In Table II, we provide the nucleation rates for all simulated bubbles alongside all parameters required for their computation. The kinetic pre-factor *A* can be obtained with the data reported in the table using Eq. (7) and is of the order of 10^{40} m⁻³ s⁻¹ for all pressures and both choices of radius definition. The rates corresponding to R_G and R_{ed} are shown as filled black and red circles, respectively, in Fig. 5. Our data are consistent with those of Ref. 20, indicated with crosses in the figure, which were obtained with Density Functional Theory for two different temperatures, namely 423 and 523 K.

3. Surface free energy and the capillarity approximation

The surface free energy, γ , between the bubbles equilibrated in the NVT Seeding simulations and the surrounding liquid can be estimated from Eq. (8). We report them in Table II for all bubbles and plot them in Fig. 6 as a function of the liquid pressure. Again, black and red symbols correspond to the Gibbs and the equi-density definitions of the critical bubble radius, respectively. Both sets of data tend to the flat interface value (blue square) obtained from our liquid-vapor direct coexistence simulations via Eq. (1). This is a good consistency test passed by the Seeding approach. Given that y is proportional to the bubble radius [see Eq. (8)] and that $R_{ed} < R_G$, γ coming from the equi-density radius definition is systematically lower. As one approaches coexistence (higher pressures/larger bubbles), both sets of data get closer to each other because the difference between both radii becomes a small fraction of the overall radius. The decrease of γ with pressure for the equidensity radius definition is about 1 nN $m^{-1}/100$ bars pressure drop, whereas it is approximately half that for the Gibbs radius. In either case, y goes down by decreasing the pressure away from coexistence. This is consistent with the results obtained in Ref. 3 at another temperature (296.4 K).

The lowering of y by moving away from coexistence is a common feature with other systems like the Lennard–Jones fluid in the cavitation/condensation transition^{28–30,33} or ice nucleation in water.^{45,67} In our case, both the curvature of the interface and the pressure of the surrounding fluid change by moving away from coexistence. Both factors simultaneously contribute to the variation of y, and their effects cannot be decoupled. Nevertheless, it seems intuitive that by lowering the pressure (or the density) of the liquid, the



406 FIG. 5. (a) Decimal logarithm of the cavitation rate as a function of the liquid pres-407 sure. Blue squares are obtained by Brute Force simulations at 450 K. Circles are 408 obtained with NVT Seeding {black circles using the Gibbs definition for the critical bubble radius [Eq. (3)] and red ones for the equi-density criterion [Eq. (4)]}. Filled 409 410 circles correspond to T = 450 K and empty ones to 550 K. The black and red 411 dashed lines are fits to the NVT Seeding data using the CNT expressions given in 412 the main text and the $\gamma(p)$ dependence shown in Fig. 6. The green (pink) dashed line represents the capillarity approximation for 450 K (550 K). The horizontal dot-413 ted lines indicate the expected rate range for cavitation in isobaric superheating¹ 414 415 and quartz inclusions isochoric cooling experiments.^{1,15} Crosses correspond to Density Functional Theory for 423 K (brown) and 523 K (purple).²⁰ (b) Enlargement 416 417 of the high nucleation rate regime.

similarity between the vapor and the liquid increases, which is in line
with a *y* decrease.

⁴²⁰ The decrease of γ with pressure looks rather linear [see ⁴²¹ Fig. 6(a)]. By fitting $\gamma(p)$ to a straight line (dashed in Fig. 6) and ⁴²² using Eqs. (5)–(8), we can fit the rate-pressure dependence²⁹ and ⁴²³ obtain the dashed curves in Fig. 5, which we comment on in more ⁴²⁴ detail in Sec. III B 4.

⁴²⁵ The variation of γ with curvature is often described by the ⁴²⁶ following expression proposed by Tolman:⁶⁵

$$\gamma = \gamma_c \left(1 - \frac{2\delta}{R_c} \right),\tag{9}$$

where δ is the Tolman length, which can be obtained from the slope of a γ vs $1/R_c$ representation such as that shown in Fig. 6(c) (filled data correspond to 450 K). We use $R_c = R_{ed}$ to do such representation given that, as it will be shown later on, it turns out that R_{ed} better predicts the nucleation rate in the spontaneous cavitation regime. A positive δ is expected, given that γ diminishes as curvature increases.



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FIG. 6. (a) and (b) Interfacial free energy as a function of the liquid pressure. Black and red symbols correspond to estimates of γ from the *NVT* Seeding simulations using Eqs. (3) and (4) to obtain the bubble radius, respectively. The blue square corresponds to the coexistence value obtained with simulations of a flat vapor–liquid interface via Eq. (1). (a) and (b) correspond to 450 and 550 K, respectively. (c) Interfacial free energy vs the inverse equi-density radius for both temperatures as indicated in the legend.

The resulting δ is 0.1 nm. Later on, in Sec. III B 5, we extend our study to a temperature of 550 K in order to compare our simulations with experiments. Here, we anticipate the results for this temperature regarding the Tolman length. From the representation in Fig. 6(c) (empty symbols), we get the same δ as for 450 K, 0.1 nm, suggesting that δ is temperature independent. A very recent simulation estimate of δ at T = 296.4 K (0.091 ± 0.008 nm) using the same

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model as in this work confirms the non-dependence of the Tolmanlength with temperature.

Taking into account, the decrease of y when lowering the liquid pressure is crucial to get accurate estimates of the nucleation rate. To illustrate this, we compute the rate that would be obtained by considering y constant and equal to its value at coexistence (capillarity approximation). To obtain such a rate, we express Eqs. (7) and (6) as a function of y via Eq. (8). Thus, one gets

 $\Delta G_c = \frac{16\pi\gamma^3}{24\pi^2},$

and

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$$A = \rho_l \sqrt{\frac{2\gamma}{\pi m}}.$$
 (11)

(10)

458 Inserting Eqs. (11) and (10) into Eq. (5) and using the coexistence 459 γ for every pressure gives rise to the green dashed curve in Fig. 5, 460 which lies at lower rates as compared to Seeding or Brute Force data. 461 The capillarity approximation actually underestimates J by 6 orders 462 of magnitude when compared to Brute Force. The conclusion is that 463 a theoretical or a simulation treatment (Seeding and Brute Force in 464 the present case) that takes into consideration the variation of γ with 465 pressure/curvature is indeed needed to obtain accurate cavitation 466 rates in overstretched water, not enough the use of the capillarity 467 approximation. It was also concluded that the capillarity approxima-468 tion fails to provide accurate nucleation rates for water cavitation, in 469 previous simulations and theoretical studies.^{2,3,}

470 4. Cavitation rate: Brute force vs Seeding

In Fig. 5 we plot the rate vs the liquid pressure as obtained from 471 472 Brute Force simulations (blue squares) and Seeding (black and red 473 circles using R_G and R_{ed} as definitions for the critical radius, respec-474 tively). More than 300 orders of magnitude of the nucleation rate are spanned by our data. Brute Force data cover the highly negative 475 476 pressure regime, where cavitation is spontaneous in the simulation 477 time, whereas Seeding is used to obtain rate estimates in a higher 478 pressure regime, where the rate is much lower and nucleation does 479 not spontaneously occur. As discussed later on, the Seeding regime 480 is more relevant to experiments.

481 The Gibbs radius systematically gives smaller rates than the 482 equi-density one (by about 4-5 orders of magnitude), given that 483 the nucleation barrier is proportional to the cubed bubble radius [Eq. (6)] and $R_G > R_{ed}$. An extrapolation to low pressures of the fits 484 to Seeding data [done by linearly extrapolating the y(p) fits shown 485 in Fig. 6(a)] suggests that by using R_{ed} as radius definition, a bet-486 487 ter consistency with Brute Force is achieved [see the enlargement 488 shown in Fig. 5(b)]. We stress that the Brute Force rate is exempt 489 from theoretical approximations. In this respect, R_{ed} seems to be a 490 better definition than R_G of the critical bubble radius, R_c , in the CNT expressions given by Eqs. (6)-(8). 491

⁴⁹² In Fig. 7, we plot the Gibbs free energy barrier, ΔG_c , com-⁴⁹³ puted according to Eq. (6), from our Seeding simulations (dots)</sup> ⁴⁹⁴ fitted via the linear $\gamma(p)$ dependence found in Fig. 6. The predicted ⁴⁹⁵ free energy barrier height in the spontaneous cavitation pressure ⁴⁹⁶ range (-750 to -650 bars) goes from 15 to 20 k_BT according to ⁴⁹⁷ the R_{ed} radius definition. These barrier heights are typical of sponta-⁴⁹⁸ neous nucleation events in simulations. ^{50,68} This consistency further



FIG. 7. Gibbs free energy as a function of the liquid pressure from Seeding simulations considering the equi-density radius definition. 450 K. Black (red) circles are data obtained using the Gibbs (equi-density) radius definition. Dashed curves are fits based on the linear $\gamma(p)$ dependence shown in Fig. 6.

demonstrates the match between Brute Force and Seeding extrapolated via the $\gamma(p)$ linear dependence of the data obtained with the equi-density radius definition.

5. Comparison with experiments

Experimental studies of cavitation, rather than having direct access to the nucleation rate, detect the thermodynamic state point at which a liquid water sample undergoes cavitation. In Fig. 8, the cavitation pressure is plotted against temperature. Empty and solid symbols correspond to experimental measurements and simulations, respectively. Lines correspond to theoretical predictions.

We compare first our data against experiments of isobaric superheating¹² and isochoric cooling, the latter performed with quartz inclusions.^{1,15} The cavitation rate probed in these experiments, J_{exp} , is determined by the product of the employed volumes and observation times and ranges from 10^{10} to 10^{15} (m⁻³ s⁻¹).^{1,15,19,69} Therefore, in order to compare our simulations against experiments, we have to find the pressure range that gives the aforementioned rates (indicated with dotted horizontal lines in Fig. 5). By using this strategy of identifying iso-rate lines and with simulations, we were able to satisfactorily predict the location of the so-called homogeneous ice nucleation line in studies of ice nucleation.⁷⁰

Using the J(p) curves in Fig. 5, we can estimate by interpolation that cavitation in the experiments mentioned in the previous paragraph will occur between $p(J = 10^{10} \text{ m}^{-3} \text{ s}^{-1}) = -41$ MPa and $p(J = 10^{15} \text{ m}^{-3} \text{ s}^{-1}) = -44$ MPa at 450 K (we used the R_{ed} radius definition for this estimate, but the values obtained using R_G are very similar). This pressure range is quite narrow due to the large slope of the J(p) curve. Such pressure range is our simulation estimate for the occurrence of cavitation in experiments of isobaric superheating or isochoric cooling (in quartz inclusions) at 450 K and is indicated with a red dot for R_{ed} and with a black dot for R_G in Fig. 8 [the size of the dot includes the values of $p(J = 10^{10}-10^{15} \text{ m}^{-3} \text{ s}^{-1})]$]. It is worth pointing out that at the pressures where we predict cavitation to occur in experiments (-41 to -44 MPa), we obtain a ΔG_c of 60–70 k_BT from the fit to the R_{ed} data shown in Fig. 7. These barrier heights are typical of nucleation experiments.

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540 FIG. 8. Cavitation pressure as a function of temperature obtained with experiments 541 (empty symbols), simulations (filled symbols), and theory (dashed lines). Black and 542 red filled dots correspond to our Seeding simulation predictions using R_G and R_{ed} as the radius definition, respectively, whereas the purple dot corresponds to transi-543 tion sampling simulations.³ The dotted line is a spline fit to simulation data to help 544 545 visualize the trend predicted by TIP4P/2005. Filled green up triangles are predictions combining simulation equations of state with the capillarity approximation 546 547 (see main text). The cyan dashed line is a CNT-like prediction from Refs. 1 and 25 548 and the maroon one is a density functional approach.²⁰ Blue¹ and pink¹⁵ empty diamonds correspond to the inclusion of isochoric cooling experiments. Empty 549 550 maroon up triangles are data from isobaric superheating experiments.¹² Right tri-551 angles correspond to Berthelot isochoric cooling experiments, in pale blue¹⁶ and 552 in pink.¹⁷ Centrifuge experiments are represented with orange down triangles, 553 shock wave experiments with blue up triangles,73 and acoustic experiments with squares, in black from Ref. 14, in red from Ref. 37, and in green from Ref. 38. The 554 555 maroon circle is obtained using artificial trees.

556 The dot in Fig. 8 obtained from our simulations at 450 K lies 557 in the low pressure limit of the rather scattered experimental data 558 cloud (blue empty diamonds) found around 450 K. This is consis-559 tent with the hypothesis that, around this temperature, there might 560 be heterogeneous cavitation in the experiments,¹ which is a pathway that leads to bubble nucleation at higher pressures than its homoge-561 562 neous counterpart. Our simulation point, obviously, corresponds to 563 homogeneous nucleation, which is why it stands close to the lowest attainable experimental pressures. 564

At high temperatures, the experimental data are not scattered 565 566 anymore, and they all nicely fall into a neat line. In order to compare 567 our simulations with experiments in such a temperature regime, we 568 performed extra NVT Seeding simulations at 550 K. The simula-569 tion details are identical to those described for 450 K except that we 570 used a lower integration time step, namely 1 fs. At 550 K, we find a coexistence pressure of 39(1) bars and an interfacial free energy of 571 572 17(2) mJ m⁻², both consistent with Ref. 61 (38.01 bars) and Ref. 62 573 $(38.3 \text{ bars and } 17.4 \text{ mJ m}^{-2})$

574 We run first a bubble of radius about 3 nm. We observe that 575 the interfacial width at 550 K doubles that of 450 K (2 vs 1 nm). 576 In equilibrium at 550 K, the bubble is surrounded by a liquid of 577 p = -73.2 bars. Since its corresponding rate, $J = 10^7 \text{ m}^{-3} \text{ s}^{-1}$ (with the 578 R_{ed} radius criterion), turned out to lie below the J range relevant to experiments, $10^{10}-10^{15}$ m⁻³ s⁻¹, we decided to equilibrate a couple 579 580 of smaller bubbles in order to enable the interpolation. The rates cal-581 culated from the three Seeding runs are reported in Table II and are 582 represented as empty circles in Fig. 5 [the kinetic pre-factor, which 583 can be obtained with the data of the table using Eq. (7), is of the same 584 order of magnitude as that found at 450 K: 10^{40} m⁻³ s⁻¹]. The $\gamma(p)$

dependence shown in Fig. 6(b) is used to fit the J(p) data at 550 K (dashed lines through empty circles in Fig. 5). By interpolation, we estimate that cavitation at 550 K will take place between -7.6 and -8.5 MPa (using the R_{ed} radius definition). This pressure interval is included as a red dot in Fig. 8 and is quite close to the experimental line, which highlights the ability of the TIP4P/2005 model, in combination with the Seeding technique, to predict and justify the experimental cavitation behavior. In Fig. 8, we also include two low temperature data coming from inclusion experiments (pink empty diamond¹⁵) and simulations (purple solid circle³). Again, simulations and inclusion experiments with each other.

To summarize, our simulations seem to be in good agreement with superheating¹² and with quartz inclusion experiments¹ at high temperatures, and those of Ref. 3 are consistent with more recent quartz inclusions experiments performed at lower temperatures.¹⁵ We include in Fig. 8 a spline fit (red dotted line) to the three available simulation data (450 and 550 K from this work and 296.4 K from Ref. 3) to give a visual idea of the cavitation line predicted by the TIP4P/2005 model. A direct comparison between the model and the experiment is enabled by the fact that TIP4P/2005 predicts quite well the critical temperature and the vapor pressure.⁶¹ The model seems to mildly overestimate the cavitation pressure, which indicates that TIP4P/2005 overestimates the nucleation rate. Such overestimation could be due to the model having a smaller y than real water.⁶² Also, it should be taken into account that γ is very sensitive to the employed cutoff and that using larger cutoff distances could improve the agreement between simulation and experiment.^{62,71} In any case, the model predictions shown in Fig. 8 are already very satisfactory.

However, the scenario is not fully clear yet: measurements of the cavitation pressure with other strategies, such as the Berthelot, centrifuge, acoustic cavitation, and artificial tree techniques, indicated with empty symbols of different colors forming a cloud located at around -25 MPa and between 0 and 25 °C, do not agree with the trend set by our simulations and those of Ref. 3. All these experiments find cavitation pressures well above those predicted by the simulations. In acoustic cavitation experiments, according to Ref. 35, the accessible rate is of the order of 10^{20} m⁻³ s⁻¹. This is 5 orders of magnitude higher than the highest value we have used in this paper (and in Ref. 3) to compare with inclusion experiments. However, if we had used the 10^{20} m⁻³ s⁻¹ rate, we would have found an even larger discrepancy because we would have predicted a lower cavitation pressure (pressure goes down as the rate goes up). Hence, we are unable to explain the discrepancy with acoustic cavitation experiments. What about Berthelot's experiments? In such experiments, isochoric cooling is performed as in quartz inclusions but in much larger volumes (about 5 orders of magnitude larger, 10^{-10} vs 10^{-15} m³).¹⁶ Also, the observation time is longer in Berthelot experiments (10⁴ vs 10 s). With these time and volume scales, rates as low as $10^5 \text{ m}^{-3} \text{ s}^{-1}$ could be probed in Berthelot experiments. Inferring the cavitation pressure from $J = 10^5 \text{ m}^{-3} \text{ s}^{-1}$, which is 5 orders of magnitude below the lower rate bound here considered, would go in the right direction, although it is not enough: both in our work at 450 K and in that of Ref. 3 at 296.4 K, one can see that, in the experimentally relevant region (that given by the dotted lines in Fig. 5), the pressure barely changes 3 MPa every 5 orders of magnitude change of the rate. This is clearly insufficient to explain the ~100 MPa

difference between simulations and Berthelot experiments shown in
 Fig. 8. Possibly, the explanation for such a large discrepancy between
 simulations and Berthelot or other sorts of experiments is that cavi tation occurs at large pressures via heterogeneous nucleation in these
 experiments.

648 Finally, in Fig. 8, we also include the capillarity approxima-649 tion prediction for the cavitation pressure in isobaric superheating or quartz inclusion experiments (green upward triangles). These tri-650 angles lie at lower pressures than circles corresponding to Seeding 651 652 predictions. This is justified as follows: We have already discussed 653 in Fig. 5 that the rate obtained from the capillarity approxima-654 tion is lower than that coming from Seeding due to the decrease 655 of γ as the bubble radius diminishes (see Fig. 6). Since the cap-656 illarity rate is lower, the pressure at which capillarity predicts a 657 certain rate is lower too. However, it is perhaps surprising that the 658 capillarity approximation prediction for the cavitation pressure is 659 quite close to that obtained by Seeding despite the fact that capil-660 larity rates lie 5-10 orders of magnitude below Seeding (see Fig. 5). 661 Again, this is explained by the steep J(p) curve in the experimen-662 tally relevant region (large rate changes correspond to rather small 663 pressure variations). This justifies the proximity between Seeding 664 and capillarity predictions of the cavitation pressure and between 665 different theoretical approaches shown in Fig. 8, disregarding^{1,25} or taking into account²⁰ the variation of the interfacial free energy with 666 pressure. 667

668 IV. SUMMARY AND CONCLUSIONS

669 We investigate water cavitation with molecular dynamics sim-670 ulations using the TIP4P/2005 model. Our first goal is to establish 671 the cavitation rate vs pressure curve at 450 K. For that purpose, we 672 first evaluate the coexistence properties by simulating the liquid in 673 contact with the vapor phase in direct coexistence. We find that the 674 coexistence pressure is 4.49 bars and that the interfacial free energy is 40(1) mJ m⁻². We then simulate the liquid at large negative pressures 675 676 until we find a regime where vapor bubbles spontaneously form in 677 the course of an unbiased simulation. Due to the small size of the 678 simulation volumes as compared to experimental ones, this regime 679 is found at very low pressures: below -650 bar. In such simulations, 680 we can estimate the nucleation rate by computing the average time 681 required for a bubble to appear in the simulated liquid volume. We find nucleation rates in the range of $10^{30} - 10^{33}$ m⁻³ s⁻¹ when lower-682 ing the pressure from -650 to -750 bar. We cannot simulate lower 683 684 pressures because there is no longer an induction period before the 685 appearance of a bubble. To compute the nucleation rate at larger 686 pressures, which are more relevant to experiments, we resort to the 687 Seeding method, which consists in simulating a bubble surrounded 688 by the liquid in the NVT ensemble, in our case at 450 K. In such sim-689 ulations, we compute the average pressure of the surrounding liquid 690 with the virial expression and the bubble radius by means of radial 691 density profiles (equi-density criterion). Alternatively, the radius is 692 estimated using the definition of the equi-molar Gibbs dividing sur-693 face, which only requires knowing the volume of the simulation box, 694 the total number of molecules, the liquid density, and the vapor 695 density. To obtain the latter, we use thermodynamic integration 696 of the equation of state of both phases, previously obtained with 697 bulk phase simulations, and impose the condition of homogeneous 698 chemical potential throughout the system. The vapor density thus

obtained is consistent with that inferred from the density profile. 699 This is expected in vapor-liquid transitions but is not generally true 700 in solid-liquid equilibrium.^{51,52} We combine the aforementioned 701 simulation information (pressure of both phases, and bubble radius) 702 with CNT to obtain estimates of the nucleation rate and the interfa-703 cial free energy. We get a huge variation of the nucleation rate (from 704 10^{-300} to 10^{10} m⁻³ s⁻¹) in the pressure range going from -130 to 705 -430 bar. The interfacial free energy goes down from its coexistence 706 value as pressure goes down. The Gibbs radius definition predicts 707 an interfacial free energy drop of ~0.5 mN/m/100 bars decrease 708 whereas the equi-density definition predicts approximately double 709 that. Both criteria to identify the bubble radius give a similar rate 710 trend although the Gibbs criterion gives lower values because it 711 predicts higher interfacial free energies. By fitting the variation of 712 the interfacial free energy with pressure to a straight line, we can 713 fit the rate data obtained with Seeding. We find that the fit to the 714 Seeding data obtained with the equi-density radius definition extrap-715 olates better to the regime where we computed the nucleation rate 716 with unbiased simulations. Thus, the equi-density radius seems to 717 be more appropriate to describe water cavitation with CNT. The γ 718 variation with curvature obtained with the equi-density radius is fit-719 720 ted by a 0.1 nm Tolman length. We also compare the Seeding rate curves with those obtained using the capillarity approximation, i.e., 721 722 by assuming that the interfacial free energy does not vary with pressure. Obviously, the capillarity curve predicts lower rates because 723 it does not take into consideration the lowering of the interfacial 724 free energy when moving away from coexistence. Finally, we com-725 726 pare our results with experiments. The experimental data available at 450 K are rather scattered so we decided to run a few extra Seeding 727 simulations at 550 K where there are cleaner data. To compare with 728 experiments, we need to predict the pressure at which the system 729 undergoes cavitation at a given temperature. We do this prediction 730 by interpolating, in our rate vs pressure curves, the rate at which it is 731 732 expected that cavitation takes place in the experiments. With this strategy, we get a successful comparison with both isobaric heat-733 734 ing and isochoric cooling experiments with small volumes (quartz inclusions). However, we find irreconcilable discrepancies with the 735 cavitation pressures obtained with other experimental techniques 736 like Berthelot or acoustic cavitation. We hypothesize that there could 737 be heterogeneous bubble nucleation in such experiments, given that 738 cavitation occurs at much higher pressures than predicted by our 739 simulations and those of Ref. 3, performed at 296.4 K. Overall, 740 our work shows an efficient manner to investigate and understand 741 742 the factors that control and affect water cavitation using molecular simulations. 743

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755 AUTHOR DECLARATIONS

756 Conflict of Interest

⁷⁵⁷ The authors have no conflicts to disclose.

758 Author Contributions

759 Cintia P. Lamas: Conceptualization (equal); Data curation (equal); 760 Formal analysis (equal); Investigation (equal); Methodology (equal); 761 Writing - original draft (supporting); Writing - review & editing 762 (equal). Carlos Vega: Conceptualization (equal); Funding acqui-763 sition (equal); Project administration (equal); Supervision (equal); Writing - review & editing (equal). Eva G. Noya: Conceptualiza-764 765 tion (equal); Data curation (equal); Project administration (equal); 766 Resources (equal); Supervision (equal); Validation (equal); Writ-767 ing - review & editing (equal). Eduardo Sanz: Conceptualization 768 (equal); Data curation (equal); Funding acquisition (equal); Method-769 ology (equal); Project administration (equal); Supervision (equal);

⁷⁷⁰ Writing – original draft (equal); Writing – review & editing (equal).

771 DATA AVAILABILITY

The data that support the findings of this study are available within the article.

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