Ice Ih vs. ice III along the homogeneous nucleation line

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Pure water can be substantially supercooled below the melting temperature. The pressures and temperatures where water is observed to freeze in supercooling experiments define the so-called homogeneous nucleation line. The slope of such a line changes from negative to positive as pressure increases. In our work, we use computer simulations with the TIP4P/Ice water model to investigate such a change of slope. The simulation prediction for the melting and the homogeneous nucleation lines is in reasonable agreement with the experiment. We find, in accordance with a previous hypothesis, that the aforementioned slope sign inversion can be explained by a change in the nucleating ice polymorph from ice Ih to ice III. Moreover, we underline the kinetic character of the homogeneous nucleation line by defining different lines for different values of the nucleation rate. We analyse in detail the factors that affect the competition between ices Ih and III in the framework of classical nucleation theory.

Understanding water freezing has strong implications in climate change prediction, cryopreservation, and planetary and atmospheric science. The simplest conceivable freezing-mechanism is the homogeneous nucleation of ice from supercooled water. Although ice formation predominantly occurs heterogeneously, it is likely that homogeneous ice nucleation from nearly pure water occurs in clean atmospheric conditions (upper troposphere). In addition, heterogeneous nucleation and crystallization from solution cannot be rationalised without having fully understood homogeneous nucleation first.

Understanding homogeneous ice nucleation in compressed water is relevant for high pressure cooling in the preservation of food and biological samples. Moreover, the freezing behaviour at high pressures can provide clues to obtain amorphous water or to probe water in the so-called no man’s land.

In 1975, Kanno, Speedy and Angell conducted a pioneering study of homogeneous ice nucleation at high pressures. By cooling emulsified microscopic water drops at different pressures, they established the so-called homogeneous nucleation line (HNL), which corresponds to the loci in the ($p,T$) plane where the drops were observed to freeze. In the absence of impurities, freezing occurs well below the melting temperature ($T_m$) because the birth of ice requires surmounting a free energy barrier due to the appearance of an interface between the emerging solid nucleus and the surrounding liquid. For instance, Kanno, Speedy and Angell found that the freezing temperature is 42 K below $T_m$ at 1 bar.

The experimental HNL is shown with a thin solid black line in Fig. 1. The black thick solid lines in the figure indicate the ice–liquid coexistence; solid and dashed lines correspond to the melting of ices Ih and III, respectively. The parallelism between the melting and the homogeneous nucleation lines suggests that the kink in the HNL corresponds to a change in...
the nucleating phase from ice Ih to ice III. This hypothesis has not been confirmed yet, because experiments do not have access to the structure of the critical crystal nucleus. The aim of this paper is to test if the HNL kink can be justified by the involvement of ice III in water freezing. This is not a trivial task. In fact, only recently, we proposed that the part of the HNL with negative slope is explained by freezing via ice Ih nuclei\(^{27}\) rather than via other metastable ice polymorphs.\(^{28}\)

We use the TIP4P/Ice water model\(^{29}\) to reach our objective. Recently, we have computed the HNL for this model considering ice Ih up to 2 kbar (corresponding to the part of the HNL with a negative slope).\(^{27}\) The model predictions for melting and HN lines, both considering ice Ih, are in fairly good agreement with the experiment (solid purple vs. solid black lines in Fig. 1). Our simulations with TIP4P/Ice enabled us to predict that the temperature difference between the ice Ih melting and HN lines increases with pressure due to an increase of the ice Ih–liquid interfacial free energy.\(^{27}\) So far, we have restricted our work to ice Ih. To test the hypothesis that the change in slope of the HNL corresponds to a change in the nucleating phase (from ice Ih to ice III), we compute the melting and the HN lines of ice III. All results in this paper regarding ice Ih have been previously published,\(^{27,30}\) whereas those corresponding to ice III are presented here for the first time.

We compute the ice III melting line by direct coexistence,\(^{31,32}\) which consists in simulating the solid and the liquid at contact in the NpT ensemble at several temperatures. Below melting the solid grows and above it melts. Thus, we enclose the melting temperature within a certain temperature range. This technique properly takes into account the orientational disorder of water molecules in the ice structure.\(^{33}\) The resulting melting line is shown as a thick purple dashed line in Fig. 1. Although the agreement with the experiment is not perfect, the model captures quite well the experimental features of the melting curves.

Next, we need to obtain the HNL for ice III. It is important to note that the position of the HNL depends on the sample volume, \(V\), and on the cooling rate.\(^{13,27}\) More specifically, the rate probed at the HNL is given by \(1/(t_c V)\),\(^{27}\) where \(t_c\) is the time spent at each Celsius degree. This ratio takes values of \(10^{16}\) m\(^{-3}\) s\(^{-1}\) in typical experiments of microscopic drops where the temperature drops a Kelvin every 10–20 seconds (\(t_c = 10–20\) s).\(^{13,27,34–42}\) Therefore, we need to evaluate the rate as a function of temperature, \(J(T)\), for different pressures in order to locate the HNL.

Because ice nucleation is a rare event, obtaining the rate as a function of pressure and temperature is a highly demanding task from a computational point of view.\(^{43}\) For instance, obtaining the ice nucleation rate of TIP4P/Ice at a single state point with Forward Flux Sampling\(^{44}\) took tens of millions of CPU hours.\(^{43}\) To locate the HNL we need the rate not just for a single state point, but for a wide range of pressures and temperatures. Therefore, we cannot use rigorous rare event techniques.\(^{44–48}\) We resort to the approximate seeding technique, that combining simulations with classical nucleation theory\(^{49–53}\) has proven able to give the correct rate trends for a variety of systems: hard spheres, Lennard-Jones, mW water and Tosi–Fumi NaCl.\(^{51,54}\) The name of “Seeding” comes from the fact that one needs to simulate ice seeds embedded in supercooled water. In previous work we computed the nucleation rate as a function of temperature for two different pressures using ice Ih seeds.\(^{27,30}\)

Here, we apply to ice III the same sort of calculations as those described in ref. 27 and 30 to obtain the nucleation rate via ice Ih seeds. The simulation details are exactly the same, so we omit them here to avoid redundancy. The only details specific to the new work we present here are the way ice III seeds were generated and detected. To build an ice III seed with proper orientational disorder we let ice III spontaneously grow in a direct coexistence simulation and cut a spherical seed from the grown solid. To detect ice III we use an averaged\(^{55}\) Steinhardt-like order parameter.\(^{56}\) In particular, we compute for each particle a \(q_6\) order parameter (see ref. 51) with a distance of 4.83 Å to identify neighbors. The \(q_6\) threshold to establish if a molecule belongs to a solid or a liquid environment weakly depends on temperature and pressure and is obtained with the mislabelling criterion as explained in ref. 51.

To obtain the ice III nucleation rate and, henceforth, the HNL we used three different seeds (of about 8000, 3000 and 500 molecules) at three different pressures. The supercooling (temperature difference with respect to melting) at which the inserted ice III nuclei were found to be critical in our seeding simulations is plotted in Fig. 2. For a given supercooling, the critical cluster size, \(N_c\), does not vary much with pressure. As we show below, this is due to the fact that, for a given supercooling, neither the chemical potential difference between the solid and the fluid nor the interfacial free energy between both phases change much with pressure for the case of ice III.

With the \(N_c\) data shown in Fig. 2 one can get fits for the nucleation rate as a function of temperature at different pressures as explained in ref. 51. In particular, we use the classical nucleation theory expression for the rate, \(J:\)\(^{24,57,58}\)

\[
J = A \exp(-\Delta G_c/(k_B T))
\]

(1)

![Fig. 2 Critical cluster size, \(N_c\) versus supercooling, \(\Delta T\), for ice III at different pressures. Points correspond to seeding simulations whereas lines are fits based on classical nucleation theory.\(^{51}\)](image-url)
where \( A \) is a kinetic pre-factor and \( \Delta G_c \) is the work required to reversibly form the critical cluster at constant \( p \) and \( T \). Assuming a spherical critical cluster, \( \Delta G_c \) is given by:

\[
\Delta G_c(T) = \frac{16\pi(\gamma(T))^3}{3(\rho_s(T))^2|\Delta \mu(T)|^2} \tag{2}
\]

where, \( \rho_s \) is the solid density, \( \gamma \) is the ice–liquid interfacial free energy and \( \Delta \mu \) is the solid–liquid chemical potential difference.

In what follows we analyse the influence of \( \rho_s \), \( \Delta \mu \), \( \gamma \) and \( A \) in the competition between both polymorphs on water freezing.

Regarding the solid density, it is about 0.93 and 1.15 g cm\(^{-3}\) for ices \( \text{Ih} \) and \( \text{III} \), respectively. Since \( \rho_s^2 \) appears in the denominator of the expression for \( \Delta G_c \) (eqn (2)), ice \( \text{III} \) will be favoured with respect to ice \( \text{Ih} \). Specifically, the barrier for nucleating ice \( \text{Ih} \) will be about 1.5 times larger due to the solid density effect.

\( |\Delta \mu| \) is obtained via thermodynamic integration from the melting temperature.\(^{51}\) For a given supercooling, \( |\Delta \mu| \) does not change significantly with either pressure or with the ice polymorph (see Fig. 3 where we show \( |\Delta \mu| \) versus \( \Delta T \) for both ice polymorphs and all pressures studied). The \( |\Delta \mu| \) curves become somewhat different at large supercooling, but this does not affect much the location of the HNLs. Then, regarding \( |\Delta \mu| \), the competition between both ice polymorphs depends on the temperature difference with respect to the corresponding melting line. In other words, unsurprisingly, the nucleation of an ice polymorph is favoured where it is the most stable phase. This explains why, as shown below, the competition between both polymorphs occurs at pressures close to the liquid–ice \( \text{Ih} \)--\( \text{III} \) triple point pressure.

We obtain \( \gamma \) via CNT as:

\[
\gamma = \left( \frac{3N_{\text{mol}} \rho_s^2 |\Delta \mu|}{32\pi} \right)^{1/3}, \tag{3}
\]

where, \( \rho_s \) is the solid density. Thus, we obtain a value of \( \gamma \) for each ice \( \text{III} \) inserted cluster. As in the previous work for ice \( \text{Ih} \),\(^{27,30,50}\) we find a linear dependence of \( \gamma \) with temperature. The linear fits to \( \gamma(T) \) for both ice polymorphs and all studied pressures are shown in Fig. 4. For a given temperature, \( \gamma \) increases with pressure for ice \( \text{Ih} \), whereas it either slightly decreases or remains constant for ice \( \text{III} \), within a 10 percent error in our \( \gamma \) estimates. Then, increasing pressure will favor ice \( \text{III} \) in what is regarded the interfacial free energy.

The kinetic pre-factor \( A \) is obtained as in the previous work via the following CNT expression:\(^{51}\)

\[
A = \rho_l \sqrt{\frac{|\Delta \mu|}{6\pi k_B T \Delta N_c}} \tag{4}
\]

where \( \rho_l \) is the liquid density and \( \gamma \) is the frequency with which molecules attach to the critical cluster. \( f^+ \) can be obtained in simulations as suggested in ref. 60. In our previous work for ice \( \text{Ih} \),\(^{27,30}\) and here for a couple of ice \( \text{III} \) clusters, we have checked that the approach proposed in ref. 60 to compute \( f^+ \) is consistent with the following CNT expression for \( \lambda \sim 1 \) molecular diameter:

\[
f^+ = \frac{24D\lambda N_c^{2/3}}{\lambda^2} \tag{5}
\]

where \( D \) is the liquid diffusion coefficient. Using the above two equations we obtain, at 2 kbar, a kinetic pre-factor \( A \) ranging from \( 10^{17.7\pm1} \) m\(^{-3}\) s\(^{-1}\) at low \( T \) to \( 10^{19.6\pm1} \) m\(^{-3}\) s\(^{-1}\) at high \( T \) for both ice polymorphs. To understand why the kinetic pre-factor is similar for both ice polymorphs we look at their ratio, which according to the expressions (3)–(5) is given by:

\[
\frac{A_{\text{III}}}{A_{\text{Ih}}} = \sqrt{\frac{\gamma_{\text{III}}}{\gamma_{\text{Ih}}}} \left( \frac{\rho_{s,\text{Ih}}}{\rho_{s,\text{III}}} \right)^{1/3} \tag{6}
\]

Since, for 2000 bar \( \gamma_{\text{III}} \) is similar to \( \gamma_{\text{Ih}} \) (see Fig. 4) and the solid densities are also comparable, it is understandable that the kinetic pre-factors are close to each other. It is expected that the analysis performed here for 2 kbar will not be that different for any other pressure in the range of 1.5–2 kbar where both polymorphs compete (close to the liquid–ice \( \text{Ih} \)--\( \text{III} \) triple pressure).
Perhaps, our data in Fig. 4 suggests that for pressures lower than 2 kbar $\gamma_{III}$ is larger than $\gamma_{Ih}$. Then, the kinetic pre-factor for ice III is expected to be higher than that of ice Ih at pressures lower than 2 kbar. However, the difference will not be larger than one order of magnitude, which is not going to significantly shift the location of the HNLs. In summary, the kinetic pre-factor does not play a key role in the competition between both ice polymorphs.

With the temperature dependence at different pressures of $\rho_s$, $|\Delta\mu|$, $\gamma$ and $A$ we can obtain fits for the nucleation rate at three different pressures. Results for ice III at 1.25, 1.6 and 2 kbar are shown in red in Fig. 5. The fits for the nucleation rate via ice Ih seeds at two different pressures are shown in blue in Fig. 5 (these results are taken from ref. 27 and 30). The horizontal line in Fig. 5 corresponds to the rate that is probed at the experimental HNL ($10^{16}$ m$^{-3}$ s$^{-1}$). Therefore, the crossings between the horizontal line and the rate curves define points of the HNL for each polymorph. The ice III 1.25 kbar rate curve does not reach $10^{16}$ m$^{-3}$ s$^{-1}$; so we only obtained points for the ice III HNL from the 1.6 and 2 kbar isobars. The HNL points thus found are plotted in Fig. 1 with purple diamonds and triangles for ices Ih and III, respectively. Although the HNL could have some curvature, lines joining these points roughly indicate where the HNL for each polymorph lies. Obviously, for a given pressure, the HNL that lies at higher temperature will determine the phase that is formed on cooling water. The ice Ih and ice III HNLs cross at around 1.7 kbar and 196 K. Therefore, the model prediction is that ice Ih will nucleate on cooling water drops below 1.7 kbar and ice III will be formed at higher pressures. The resemblance between the simulation and the experimental HNLs is quite evident. This suggests that, indeed, the change in slope in the experimental HNL is due to a change in the nucleating phase, which is the main conclusion we draw from our study.

A due comment here is that the HNL is a kinetic and not a thermodynamic line. The HNL defined so far corresponds to the loci in the ($p$–$T$) plane where the rate is $10^{16}$ m$^{-3}$ s$^{-1}$. We now define another homogeneous nucleation line, HNL$_{100}$, as that given by a rate of $10^{100}$ m$^{-3}$ s$^{-1}$. Although such rate is not accessible experimentally, defining the HNL$_{100}$ serves for our purpose of illustrating that the freezing line is indeed a kinetic one. This academic exercise requires obtaining $\dot{f}(T)$ for a wide temperature range, a task that is nowadays only accessible to seeding simulations. The HNL$_{100}$ is shown in turquoise in Fig. 1 (diamonds joined by solid lines correspond to nucleation via ice Ih whereas triangles joined by dashed lines correspond to ice III). The HNL$_{100}$ is significantly shifted with respect to the HNL defined for $J = 10^{16}$ m$^{-3}$ s$^{-1}$ (in purple), which shows the kinetic character of the freezing line. The HNL$_{100}$ for ice III and ice Ih cross at 1.4 kbar and 227 K (versus 1.7 kbar and 196 K for $J = 10^{16}$ m$^{-3}$ s$^{-1}$). Obviously, the HNL$_{100}$ is shifted to higher temperatures because ice forms at lower rates in warmer water. The shift of the crossing point from 1.7 to 1.4 kbar is not so obvious because, as discussed above, the competition between both polymorphs depends on a balance between densities, chemical potentials and interfacial free energies.

In summary, we have used the seeding technique to evaluate the HNL for the TIP4P/Ice water model. Both the melting and the HN lines of the model are in reasonable quantitative agreement with the real ones. As pressure increases, the slope of the HNL changes from negative to positive at a pressure close to that of the liquid–ice Ih–ice III triple point. This slope switch corresponds to a change in the structure of the emerging nucleus from ice Ih to ice III, which is the main conclusion of our work. We show that the location of the HNL depends on the nucleation rate for which the HNL is defined. This underlines the kinetic character of the HNL. To conclude, we summarize the factors that affect the competition between both polymorphs as follows: the kinetic pre-factor is not decisive, the solid density favors ice III at any pressure, the chemical potential difference favors the stable phase and pressure favors ice III because its interfacial free energy does not vary much with pressure whereas that of ice Ih increases with pressure.

Conflicts of interest

There are no conflicts to declare.

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